# LOCKING TECHNIQUES FOR RF OSCILLATORS AT 5 - 6 GHZ FREQUENCY RANGE

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# ABSTRACT

This work aims at analyzing three different techniques for synchronizing RF oscillators. These techniques are Injection Locking (ILO), Phase Locked Loop (PLL) and Injection Locked Phase Locked Loop (ILPLL). ILPLL, which is a combination of PLL and ILO, has superior noise performance –compared to all the rest- at medium frequency offsets and the same noise performance at low and high offsets. Furthermore, the ILPLL has better locking range and lower phase noise than the ILO for phase-shifts close to  $\pm 90^{\circ}$ . In this work we present two different approaches for the study of the performance of the ILPLL, which as we show, produce equivalent results concerning the noise. A common gate VCO was used and the injected signal was the same for comparison reasons.

## 1. INTRODUCTION

An RF local oscillator, which usually has poor phase noise performance, can be stabilized to a reference signal by various techniques. Using the ILO technique a signal with high frequency stability is injected into the noisy oscillator, so the output will have the phase noise performance of the reference at low frequency offsets (near carrier) and the phase noise performance of the noisy oscillator at high frequency offsets. At medium frequency offsets the phase noise is confined to low levels according to the noise performance of the system, as presented in the following sections. The disadvantage of the ILO compared to a PLL is the small locking range. Further improvement in noise performance could be achieved if we modify the circuit such that a feedback loop consisted of a phase detector and a filter will connect the output of the ILO to the varactor of the VCO. The reference signal, which is injected to the VCO, is injected to the phase detector (PD) also. This circuit is called Injection Locked Phase Locked Loop (ILPLL). During recent years several researchers have been involved in the study of the ILPLL [1-5], which demonstrates superior noise performance, high output power and wider locking bandwidth compared to the ILO. Depending on the filter parameters the phase noise of the ILPLL could be further reduced at any frequency offset from carrier.

In this work an RF oscillator is first demonstrated. The equations for the locking bandwidth and the phase noise of an ILO are then derived and compared to these of a simple PLL. Two different approaches for the noise performance of the ILPLL are presented. Although ILPLL noise behavior has been studied in the past ([1],[3]) there is no systematic treatment in terms of a general PLL function block. One of these approaches is from the known literature. In the other one we demonstrate that the ILPLL can be considered as a PLL in which the phase noise of an ILO replaces the phase noise of the VCO in the PLL noise expression. Using this model we derive the noise performance of the ILPLL. It is shown that by careful design of the filter this technique has superior performance.

## 2. RF OSCILLATOR

To implement the RF voltage controlled oscillator, we choose a negative resistance topology with a low noise FET in a common gate configuration and a varactor diode. The circuit and the components we used are shown in Fig. 1. The tunable resonator between the gate and the ground (inductor with a varactor termination) and the capacitive termination of the source creates an effective negative resistance at resonance.



Figure 1. The negative resistance VCO.

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The above circuit was simulated using ADS  $2002^{\circ}$  and the resulting tuning range of the oscillator was found equal to 1.3 GHz, from 5.1 GHz to 6.4 GHz as shown in Fig. 2.



The phase noise of the free-running VCO at 5.2 GHz is the dashed line curve shown in Fig. 3. The tuning sensitivity is  $K_0 = 90$  MHz/V, as determined from Fig. 2.

#### 3. INJECTION LOCKING

Injection locking was first examined by Van der Pol, and later by Kaneyuki Kurokawa. The main principle is based on the fact that any oscillator will be synchronized to an externally applied injected signal, if the free-running frequency is close to the signal's frequency. The phase of the injected oscillator is [6]

$$\frac{da}{dt} = -\frac{V_{inj}}{V_0} \frac{\omega_0}{2Q} \sin a + \Delta \omega_0 \tag{1}$$

Half the entire locking range is :

$$\Delta \omega_{lock} = \frac{\omega_o}{2Q} \frac{V_{inj}}{V_o}$$
(2)

where  $V_0$  is the amplitude of the free-running signal,  $V_{ini}$ 

is the amplitude of the injected signal, Q is the quality factor of the tuning network and  $\omega_0$  is the instantaneous free-running frequency. Low Q is necessary for wide locking range, while high Q is required for low phase noise (Leeson's equation) and for the maintenance of the oscillation.

Using equation (1) it can be easily shown that the VCO will lock to the injected signal if

$$\frac{\omega_o}{2Q} \frac{V_{inj}}{V_o} > |\Delta\omega_0| \tag{3}$$

The phase noise of the ILO (power spectral density) is given by [1], [2]

$$S_{ILO} = \frac{\left(\frac{\omega_o}{2Q} \frac{V_{inj}}{V_o}\right)^2 \cdot \cos^2(\varphi) \cdot S_{inj} + \omega^2 \cdot S_{VCO}}{\left(\frac{\omega_o}{2Q} \frac{V_{inj}}{V_o}\right)^2 \cdot \cos^2(\varphi) + \omega^2}$$
(4)

where  $\phi$  is the phase difference between the instantaneous output phase and the phase of the injected

signal and  $S_{VCO}$  is the phase noise of the free-running VCO. In Fig. 3 the phase noise of the ILO is shown for a specific injected signal. Phase shift  $\varphi$  is close to  $0^0$  at this case and  $V_{inj}/V_0 = 0.3$ . Higher  $V_{inj}/V_0$  results in lower phase noise and phase shifts close to  $\pm 90^0$  produce higher phase noise for the ILO.

For a resonance at 5.8 GHz the synchronizing range is about  $\pm$  150MHz. The above expression is the same as the one given by Kurokawa. The phase noise at sections 3, 4 and 5 (ILO, PLL, ILPLL) was calculated using MATLAB 5.3



Figure 3. Phase noise of the ILO, Free running VCO and Injected Signal.

## 4. PHASE LOCKED LOOP

The general block diagram of a PLL is shown in Fig. 4, where  $S_{REF}$  is the phase noise of the input reference signal,  $S_{VCO}$  is the phase noise of the free-running VCO,  $S_{PD}$  is the noise of the phase detector and  $S_{LF}$  is the noise of the loop filter.



Figure 4. Fundamental PLL.

It can be easily derived that the total phase noise of the PLL is:

$$S_{PLL} = S_{REF} |H(s)|^{2} + |1 - H(s)|^{2} S_{VCO} + \left| \{1 - H(s)\} \frac{K_{VCO}}{s} F(s) \right|^{2} S_{PD}$$
(5)

where H(s) is the close-loop transfer function of the PLL. In this formula the contribution to the total noise of the loop filter is considered insignificant. A classical active loop filter topology was used. Using the same reference signal and the same VCO as in section 3 and for zero PD noise, the phase noise of the PLL is as shown in Fig. 5.

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Signal.

## 5. INJECTION LOCKED PHASE LOCK LOOP

Injection Locked Phase Locked Loop (ILPLL), a combination of ILO and PLL mechanisms, is one of the most often used techniques in optical and recently in electronic systems. We can view the ILPLL in two ways: As an ILO in which the phase noise of a PLL is used in place of the phase noise of the VCO (Fig. 6a). This structure is discussed in [1]. Using another approach we will demonstrate that ILPLL is a PLL in which the phase noise of an ILO replaces the phase noise of the VCO in the PLL noise equation (5). Fig. 6b is the corresponding block diagram.



Figure 6. The two different structures of the ILPLL.

Below we will show that these two structures are equivalent.

If we replace the term  $\omega^2 S_{\nu CO}$  in (4) by  $\omega^2 S_{PLL}$  equation (4) becomes:

$$S_{ILPLL} = \frac{\left(\frac{\omega_o}{2Q} \frac{V_{inj}}{V_o}\right)^2 \cdot \cos^2(\varphi) \cdot S_{inj} + \omega^2 \cdot S_{PLL}}{\left(\frac{\omega_o}{2Q} \frac{V_{inj}}{V_o}\right)^2 \cdot \cos^2(\varphi) + \omega^2}$$
(6)

Let us consider the system shown in Fig 6b. If  $\delta \tilde{\vartheta}$  is the Fourier transform of small phase fluctuations  $\delta \vartheta$  then we get:

$$\delta \widetilde{9}_{OUT} = \frac{\delta \widetilde{9}_{ILO}}{1 + K_{PD} \frac{K_{VCO}}{s} F(s)} + \frac{\frac{K_{VCO}}{s} \{\delta \widetilde{v}_{LF} + \delta \widetilde{v}_{PD} F(s)\}}{1 + K_{PD} \frac{K_{VCO}}{s} F(s)} + \frac{\delta \widetilde{9}_{REF} K_{PD} \frac{K_{VCO}}{s} F(s)}{1 + K_{PD} \frac{K_{VCO}}{s} F(s)}$$
(7)

 $\delta \tilde{v}_{PD}$  and  $\delta \tilde{v}_{LF}$  correspond to the phase detector noise and the loop filter noise respectively. It can be easily derived that the total phase noise of the PLL,  $S_{ILPLL}$ , is the ensemble average of the product  $\delta \tilde{9}_{OUT} \cdot \delta \tilde{9}_{OUT}^{*}$ , where \* stands for the complex conjugate operation. Assuming that the contribution to the total phase noise of the loop filter is insignificant, and all spectral variables are uncorrelated except from  $\delta \tilde{9}_{ILO}$  and  $\delta \tilde{9}_{REF}$ , we have:

$$S_{ILPLL} = S_{REF} |H(s)|^{2} + |1 - H(s)|^{2} S_{ILO}$$
  
+  $Y |1 - H(s)|^{2} K_{VCO} K_{PD} S_{REF}$  (8)

where Y is a constant given by:

$$\frac{\left(\frac{\omega_{o}}{2Q}\frac{V_{inj}}{V_{o}}\right)\cos\varphi}{\left(\frac{\omega_{o}}{2Q}\frac{V_{inj}}{V_{o}}\right)\cos\varphi+j\omega}\cdot\frac{F^{*}(s)}{s^{*}}+\left(\frac{\left(\frac{\omega_{o}}{2Q}\frac{V_{inj}}{V_{o}}\right)\cos\varphi}{\left(\frac{\omega_{o}}{2Q}\frac{V_{inj}}{V_{o}}\right)\cos\varphi+j\omega}\right)\cdot\frac{F(s)}{s}$$

 $Y \le 1$  for typical values of  $\tau_1$ ,  $\tau_2$ . Furthermore  $S_{ILO} >> S_{REF}$  at high frequency offsets so the last term in (8) can be neglected. Without the last term, equation (8) is the well-known equation for the PLL in which the phase noise of the ILO substitutes the phase noise of the free-running VCO.

Using the results from previous sections and the same active filter we plot the phase noise of the ILPLL using both equations (6) and (8). Substituting (4) to (8) and (5) to (6) and considering that  $S_{REF} = S_{ini}$  we get:

$$S_{ILPLL} = S_{REF} |H(s)|^{2} + |1 - H(s)|^{2} \frac{\left(\frac{\omega_{o}}{2Q} \frac{V_{inj}}{V_{o}}\right)^{2} \cdot \cos^{2}(\varphi) \cdot S_{inj} + \omega^{2} \cdot S_{VCO}}{\left(\frac{\omega_{o}}{2Q} \frac{V_{inj}}{V_{o}}\right)^{2} \cdot \cos^{2}(\varphi) + \omega^{2}}$$
(9)

and

$$S_{ILPLL} = \frac{\left(\frac{\omega_{o}}{2Q} \frac{V_{inj}}{V_{o}}\right)^{2} \cdot \cos^{2}(\varphi) \cdot S_{inj}}{\left(\frac{\omega_{o}}{2Q} \frac{V_{inj}}{V_{o}}\right)^{2} \cdot \cos^{2}(\varphi) + \omega^{2}} + \frac{\omega^{2} \{S_{REF} |H(s)|^{2} + |1 - H(s)|^{2} S_{VCO}\}}{\left(\frac{\omega_{o}}{2Q} \frac{V_{inj}}{V_{o}}\right)^{2} \cdot \cos^{2}(\varphi) + \omega^{2}}$$
(10)

Equation (10) is the well-known equation for the ILO in which the phase noise of the PLL substitutes the phase noise of the VCO. It is not obvious that the two equations (9), (10) produce the same results. However, considering that the terms associated to  $S_{REF}$  and  $S_{inj}$  are dominant at low frequency offsets and the terms associated to  $S_{VCO}$  are dominant at high frequency offsets, these equations generate the same results.



Figure 7. ILPLL (Fig. 6a) Phase Noise.

All the above results are valid when  $\varphi$  is close to  $0^0$ . When  $\varphi$  is close to  $\pm 90^0$  the contribution of the phase noise of the free-running VCO increases the overall phase noise of the ILO according to equation (4).



Figure 8. ILPLL (Fig. 6b) Phase Noise.

For the ILPLL and when  $\varphi$  is close to  $\pm 90^{\circ}$  the contribution of the phase noise of the PLL is prevalent, which is significantly lower than the phase noise of the VCO. This is another advantage of the ILPLL technique. To estimate the locking range of the ILPLL we use the well-known equation for the PLL

$$\omega_r = \omega_0 + K_{VCO} K_{PD} K_{LF} \cos(\vartheta_r - \vartheta_0)$$
(11)

and equation (1), while setting  $\frac{da}{dt}$  equal to zero

$$\omega_{inj} = \omega_0 + \frac{V_{inj}}{V_0} \frac{\omega_0}{2Q} \sin(\vartheta_r - \vartheta_0)$$
(12)

Taking the approach that the ILPLL is a PLL in which the phase noise of an ILO replaces the phase noise of the VCO, we substitute  $\omega_{inj}$  from (12) into  $\omega_0$  in (11) and after some mathematical manipulations we get:

$$\omega_r = \omega_0 + \Delta \omega \sin(\vartheta_r - \vartheta_0 + \varphi)$$
 (13)

where

$$\Delta \omega = \sqrt{\left(\frac{V_{inj}}{V_0} \frac{\omega_0}{2Q}\right)^2 + \left(K_{VCO} K_{PD} K_{LF}\right)^2}$$
(14)

and

$$\tan \varphi = \frac{K_{VCO}K_{PD}K_{LF}}{\frac{V_{inj}}{V_0}\frac{\omega_0}{2Q}}$$
(15)

Considering that the ILPLL is an ILO, in which the phase noise of a PLL is used in place of the phase noise of the VCO,  $\omega_r$  is substituted into  $\omega_0$  and the same equation (13) resulted. From (14) it is obvious that the ILPLL has improved locking range compared to the ILO, similar to that of the PLL.

### 6. CONCLUSIONS

We investigated the phase noise performance and the locking range of an ILO, a PLL and an ILPLL. The results present a definite advantage of the ILPLL mechanism. Further phase noise reduction (with respect to the PLL and the ILO) at medium frequency offsets and wider locking range are the main benefits. Both characteristics are analysed by using two different methods providing the same results. One of them is from the known literature, while in the other the ILPLL is treated as a PLL in which the phase noise of an ILO replaces the phase noise of the VCO.

By using a low-power, stable signal as the reference we can produce a wideband, high-power signal, which maintains the phase noise characteristics of the reference. As a result the noise power at medium frequency offsets is considerably reduced.

## 7. REFERENCES

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